Comparar modelos Polinómico vs Regresión Lineal Simple

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# Objetivo

Evaluar modelos polinómico VS modelo Regresión lineal simple con un conjunto de datos para realizar predicciones.

# Descripción

* ¿Qué hay que evaluar en los modelos
* Cargar Datos
* Describir los datos
* Limpiar datos
* Dividir datos entrenamiento y validación
* Construir modelo regresión lineal simple
* Construir modelo polinómico
* Evaluar modelos
  + R Square. Determinar cuanto explica la variable independiente a la variable dependiente. ¿Explicar por encima del 70%?
  + Residual Standar Error decsribir cual modelo es más confiable, aquel que tenga valor mas cercano a cero
  + La confianza de los predictores con el valor de la probabilidad pr(t). *,* ***,*** . Si son estadístiacmente significativos y tanto: 0.90, 0.95, 0.99.
* Predicciones con ambos modelos
* Comparar predicciones
* Iterpretar el ejercicio

# Librerías

library(ggplot2) # Visualizar datos  
library(caret) # Partir datos  
library(dplyr) # select filter mutate  
library(readr) # Leer datos csv  
# library()

# Cargar datos

## Cargar datos localmente

datos <- read.csv("../../datos/datos.FIFA.limpios.csv")

## Cargar datos de la WEB csv

# datos <- read.csv("https://raw.githubusercontent.com/rpizarrog/CIIT.-Diplomado-en-Ciencia-de-los-Datos-e-IoT/main/M%C3%B3dulo%20II/datos/datos.FIFA.limpios.csv", encoding = "iso-8859-1")

# Describir los datos

str(datos)

## 'data.frame': 17955 obs. of 50 variables:  
## $ X : int 1 2 3 4 5 6 7 8 9 10 ...  
## $ Name : chr "L. Messi" "Cristiano Ronaldo" "Neymar Jr" "De Gea" ...  
## $ Age : int 31 33 26 27 27 27 32 31 32 25 ...  
## $ Nationality : chr "Argentina" "Portugal" "Brazil" "Spain" ...  
## $ Overall : int 94 94 92 91 91 91 91 91 91 90 ...  
## $ Potential : int 94 94 93 93 92 91 91 91 91 93 ...  
## $ Club : chr "FC Barcelona" "Juventus" "Paris Saint-Germain" "Manchester United" ...  
## $ Preferred.Foot : chr "Left" "Right" "Right" "Right" ...  
## $ International.Reputation: int 5 5 5 4 4 4 4 5 4 3 ...  
## $ Weak.Foot : int 4 4 5 3 5 4 4 4 3 3 ...  
## $ Skill.Moves : int 4 5 5 1 4 4 4 3 3 1 ...  
## $ Height : chr "5'7" "6'2" "5'9" "6'4" ...  
## $ Weight : chr "159lbs" "183lbs" "150lbs" "168lbs" ...  
## $ Crossing : int 84 84 79 17 93 81 86 77 66 13 ...  
## $ Finishing : int 95 94 87 13 82 84 72 93 60 11 ...  
## $ HeadingAccuracy : int 70 89 62 21 55 61 55 77 91 15 ...  
## $ ShortPassing : int 90 81 84 50 92 89 93 82 78 29 ...  
## $ Volleys : int 86 87 84 13 82 80 76 88 66 13 ...  
## $ Dribbling : int 97 88 96 18 86 95 90 87 63 12 ...  
## $ Curve : int 93 81 88 21 85 83 85 86 74 13 ...  
## $ FKAccuracy : int 94 76 87 19 83 79 78 84 72 14 ...  
## $ LongPassing : int 87 77 78 51 91 83 88 64 77 26 ...  
## $ BallControl : int 96 94 95 42 91 94 93 90 84 16 ...  
## $ Acceleration : int 91 89 94 57 78 94 80 86 76 43 ...  
## $ SprintSpeed : int 86 91 90 58 76 88 72 75 75 60 ...  
## $ Agility : int 91 87 96 60 79 95 93 82 78 67 ...  
## $ Reactions : int 95 96 94 90 91 90 90 92 85 86 ...  
## $ Balance : int 95 70 84 43 77 94 94 83 66 49 ...  
## $ ShotPower : int 85 95 80 31 91 82 79 86 79 22 ...  
## $ Jumping : int 68 95 61 67 63 56 68 69 93 76 ...  
## $ Stamina : int 72 88 81 43 90 83 89 90 84 41 ...  
## $ Strength : int 59 79 49 64 75 66 58 83 83 78 ...  
## $ LongShots : int 94 93 82 12 91 80 82 85 59 12 ...  
## $ Aggression : int 48 63 56 38 76 54 62 87 88 34 ...  
## $ Interceptions : int 22 29 36 30 61 41 83 41 90 19 ...  
## $ Positioning : int 94 95 89 12 87 87 79 92 60 11 ...  
## $ Vision : int 94 82 87 68 94 89 92 84 63 70 ...  
## $ Penalties : int 75 85 81 40 79 86 82 85 75 11 ...  
## $ Composure : int 96 95 94 68 88 91 84 85 82 70 ...  
## $ Marking : int 33 28 27 15 68 34 60 62 87 27 ...  
## $ StandingTackle : int 28 31 24 21 58 27 76 45 92 12 ...  
## $ SlidingTackle : int 26 23 33 13 51 22 73 38 91 18 ...  
## $ GKDiving : int 6 7 9 90 15 11 13 27 11 86 ...  
## $ GKHandling : int 11 11 9 85 13 12 9 25 8 92 ...  
## $ GKKicking : int 15 15 15 87 5 6 7 31 9 78 ...  
## $ GKPositioning : int 14 14 15 88 10 8 14 33 7 88 ...  
## $ GKReflexes : int 8 11 11 94 13 8 9 37 11 89 ...  
## $ Valor : int 110500000 77000000 118500000 72000000 102000000 93000000 67000000 80000000 51000000 68000000 ...  
## $ Estatura : num 1.7 1.88 1.75 1.93 1.8 1.73 1.73 1.83 1.83 1.88 ...  
## $ PesoKgs : num 72.1 83 68 76.2 69.8 ...

summary(datos)

## X Name Age Nationality   
## Min. : 1 Length:17955 Min. :16.0 Length:17955   
## 1st Qu.: 4490 Class :character 1st Qu.:21.0 Class :character   
## Median : 8978 Mode :character Median :25.0 Mode :character   
## Mean : 8978 Mean :25.1   
## 3rd Qu.:13466 3rd Qu.:28.0   
## Max. :17955 Max. :45.0   
##   
## Overall Potential Club Preferred.Foot   
## Min. :46.00 Min. :48.00 Length:17955 Length:17955   
## 1st Qu.:62.00 1st Qu.:67.00 Class :character Class :character   
## Median :66.00 Median :71.00 Mode :character Mode :character   
## Mean :66.23 Mean :71.32   
## 3rd Qu.:71.00 3rd Qu.:75.00   
## Max. :94.00 Max. :95.00   
##   
## International.Reputation Weak.Foot Skill.Moves Height   
## Min. :1.000 Min. :1.000 Min. :1.000 Length:17955   
## 1st Qu.:1.000 1st Qu.:3.000 1st Qu.:2.000 Class :character   
## Median :1.000 Median :3.000 Median :2.000 Mode :character   
## Mean :1.114 Mean :2.947 Mean :2.363   
## 3rd Qu.:1.000 3rd Qu.:3.000 3rd Qu.:3.000   
## Max. :5.000 Max. :5.000 Max. :5.000   
## NA's :48 NA's :48 NA's :48   
## Weight Crossing Finishing HeadingAccuracy  
## Length:17955 Min. : 5.00 Min. : 2.00 Min. : 4.0   
## Class :character 1st Qu.:38.00 1st Qu.:30.00 1st Qu.:44.0   
## Mode :character Median :54.00 Median :49.00 Median :56.0   
## Mean :49.75 Mean :45.59 Mean :52.3   
## 3rd Qu.:64.00 3rd Qu.:62.00 3rd Qu.:64.0   
## Max. :93.00 Max. :95.00 Max. :94.0   
## NA's :48 NA's :48 NA's :48   
## ShortPassing Volleys Dribbling Curve   
## Min. : 7.00 Min. : 4.00 Min. : 4.00 Min. : 6.00   
## 1st Qu.:54.00 1st Qu.:30.00 1st Qu.:49.00 1st Qu.:34.00   
## Median :62.00 Median :44.00 Median :61.00 Median :49.00   
## Mean :58.72 Mean :42.94 Mean :55.42 Mean :47.22   
## 3rd Qu.:68.00 3rd Qu.:57.00 3rd Qu.:68.00 3rd Qu.:62.00   
## Max. :93.00 Max. :90.00 Max. :97.00 Max. :94.00   
## NA's :48 NA's :48 NA's :48 NA's :48   
## FKAccuracy LongPassing BallControl Acceleration   
## Min. : 3.00 Min. : 9.00 Min. : 5.00 Min. :12.00   
## 1st Qu.:31.00 1st Qu.:43.00 1st Qu.:54.00 1st Qu.:57.00   
## Median :41.00 Median :56.00 Median :63.00 Median :67.00   
## Mean :42.88 Mean :52.73 Mean :58.42 Mean :64.62   
## 3rd Qu.:57.00 3rd Qu.:64.00 3rd Qu.:69.00 3rd Qu.:75.00   
## Max. :94.00 Max. :93.00 Max. :96.00 Max. :97.00   
## NA's :48 NA's :48 NA's :48 NA's :48   
## SprintSpeed Agility Reactions Balance   
## Min. :12.00 Min. :14.00 Min. :21.00 Min. :16.00   
## 1st Qu.:57.00 1st Qu.:55.00 1st Qu.:56.00 1st Qu.:56.00   
## Median :67.00 Median :66.00 Median :62.00 Median :66.00   
## Mean :64.74 Mean :63.54 Mean :61.82 Mean :63.97   
## 3rd Qu.:75.00 3rd Qu.:74.00 3rd Qu.:68.00 3rd Qu.:74.00   
## Max. :96.00 Max. :96.00 Max. :96.00 Max. :96.00   
## NA's :48 NA's :48 NA's :48 NA's :48   
## ShotPower Jumping Stamina Strength   
## Min. : 2.00 Min. :15.00 Min. :12.00 Min. :17.00   
## 1st Qu.:45.00 1st Qu.:58.00 1st Qu.:56.00 1st Qu.:58.00   
## Median :59.00 Median :66.00 Median :66.00 Median :67.00   
## Mean :55.49 Mean :65.12 Mean :63.22 Mean :65.33   
## 3rd Qu.:68.00 3rd Qu.:73.00 3rd Qu.:74.00 3rd Qu.:74.00   
## Max. :95.00 Max. :95.00 Max. :96.00 Max. :97.00   
## NA's :48 NA's :48 NA's :48 NA's :48   
## LongShots Aggression Interceptions Positioning Vision   
## Min. : 3.00 Min. :11.00 Min. : 3.00 Min. : 2 Min. :10.00   
## 1st Qu.:33.00 1st Qu.:44.00 1st Qu.:26.00 1st Qu.:39 1st Qu.:44.00   
## Median :51.00 Median :59.00 Median :52.00 Median :55 Median :55.00   
## Mean :47.13 Mean :55.88 Mean :46.69 Mean :50 Mean :53.45   
## 3rd Qu.:62.00 3rd Qu.:69.00 3rd Qu.:64.00 3rd Qu.:64 3rd Qu.:64.00   
## Max. :94.00 Max. :95.00 Max. :92.00 Max. :95 Max. :94.00   
## NA's :48 NA's :48 NA's :48 NA's :48 NA's :48   
## Penalties Composure Marking StandingTackle   
## Min. : 5.00 Min. : 3.00 Min. : 3.00 Min. : 2.00   
## 1st Qu.:39.00 1st Qu.:51.00 1st Qu.:30.00 1st Qu.:27.00   
## Median :49.00 Median :60.00 Median :53.00 Median :55.00   
## Mean :48.55 Mean :58.65 Mean :47.26 Mean :47.68   
## 3rd Qu.:60.00 3rd Qu.:67.00 3rd Qu.:64.00 3rd Qu.:66.00   
## Max. :92.00 Max. :96.00 Max. :94.00 Max. :93.00   
## NA's :48 NA's :48 NA's :48 NA's :48   
## SlidingTackle GKDiving GKHandling GKKicking GKPositioning   
## Min. : 3.00 Min. : 1.00 Min. : 1.00 Min. : 1.0 Min. : 1.00   
## 1st Qu.:24.00 1st Qu.: 8.00 1st Qu.: 8.00 1st Qu.: 8.0 1st Qu.: 8.00   
## Median :52.00 Median :11.00 Median :11.00 Median :11.0 Median :11.00   
## Mean :45.64 Mean :16.59 Mean :16.37 Mean :16.2 Mean :16.36   
## 3rd Qu.:64.00 3rd Qu.:14.00 3rd Qu.:14.00 3rd Qu.:14.0 3rd Qu.:14.00   
## Max. :91.00 Max. :90.00 Max. :92.00 Max. :91.0 Max. :90.00   
## NA's :48 NA's :48 NA's :48 NA's :48 NA's :48   
## GKReflexes Valor Estatura PesoKgs   
## Min. : 1.00 Min. : 10000 Min. :1.550 Min. : 49.90   
## 1st Qu.: 8.00 1st Qu.: 325000 1st Qu.:1.750 1st Qu.: 69.85   
## Median :11.00 Median : 700000 Median :1.800 Median : 74.84   
## Mean :16.68 Mean : 2444530 Mean :1.812 Mean : 75.28   
## 3rd Qu.:14.00 3rd Qu.: 2100000 3rd Qu.:1.850 3rd Qu.: 79.83   
## Max. :94.00 Max. :118500000 Max. :2.060 Max. :110.22   
## NA's :48 NA's :48 NA's :48

## Variables de interés. Pregunta de interés.

¿Cual es la predicción del peso de acuerdo a la altura de un jugador de fútbol internacional?

Aplicar modelo de regresión lineal simple Aplicar modelo de regresión polinómico

* Altura y el peso. Estatura y Pesokgs

## Datos filtrados

datos <- select(datos, Estatura, PesoKgs)

# Limpiar datos

Los datos vienen con valores de NA en algunos registros, Quitarlos.

cuales <- subset(datos, is.na(Estatura))  
cuales <- rownames(cuales)  
cuales <- as.numeric(cuales)  
cuales

## [1] 13036 13037 13038 13039 13040 13041 13042 13043 13044 13045 13046 13047  
## [13] 13048 13049 13050 13051 13052 13053 13054 13055 13056 13057 13058 13059  
## [25] 13060 13061 13062 13063 13064 13065 13066 13067 13068 13069 13070 13071  
## [37] 13072 13073 13074 13075 13076 13077 13078 13079 13080 13081 13082 13083

# Quitar registros  
datos <- datos[-cuales, ]

## Verificar que no hay valores NA

summary(datos)

## Estatura PesoKgs   
## Min. :1.550 Min. : 49.90   
## 1st Qu.:1.750 1st Qu.: 69.85   
## Median :1.800 Median : 74.84   
## Mean :1.812 Mean : 75.28   
## 3rd Qu.:1.850 3rd Qu.: 79.83   
## Max. :2.060 Max. :110.22

## head(datos)

head(datos)

## Estatura PesoKgs  
## 1 1.70 72.12  
## 2 1.88 83.01  
## 3 1.75 68.04  
## 4 1.93 76.20  
## 5 1.80 69.85  
## 6 1.73 73.94

## tail(datos)

tail(datos)

## Estatura PesoKgs  
## 17950 1.85 76.20  
## 17951 1.75 60.78  
## 17952 1.91 77.11  
## 17953 1.73 67.13  
## 17954 1.78 69.85  
## 17955 1.78 79.83

# Crear datos de entrenamiento y validacón

set.seed(2022)  
entrena <- createDataPartition(y = datos$PesoKgs, p = 0.70, list = FALSE, times = 1)  
  
# entrena # Números d registro  
  
# Datos entrenamiento  
datos.entrenamiento <- datos[entrena, ] # [renglones, columna]  
  
# Datos validación  
datos.validacion <- datos[-entrena, ]  
#datos

## head(datos.entrenamiento, 10)

head(datos.entrenamiento, 10)

## Estatura PesoKgs  
## 1 1.70 72.12  
## 2 1.88 83.01  
## 3 1.75 68.04  
## 4 1.93 76.20  
## 5 1.80 69.85  
## 8 1.83 86.18  
## 9 1.83 82.10  
## 10 1.88 87.09  
## 12 1.83 76.20  
## 14 1.73 67.13

## tail(datos.entrenamiento, 10)

tail(datos.entrenamiento, 10)

## Estatura PesoKgs  
## 17941 1.85 74.84  
## 17942 1.73 66.22  
## 17944 1.73 63.96  
## 17945 1.70 66.22  
## 17946 1.78 63.96  
## 17948 1.68 68.04  
## 17950 1.85 76.20  
## 17951 1.75 60.78  
## 17953 1.73 67.13  
## 17954 1.78 69.85

## head(datos.validacion, 10)

head(datos.validacion, 10)

## Estatura PesoKgs  
## 6 1.73 73.94  
## 7 1.73 66.22  
## 11 1.83 79.83  
## 13 1.88 78.02  
## 15 1.68 72.12  
## 24 1.73 69.85  
## 26 1.78 73.03  
## 31 1.75 78.93  
## 33 1.73 68.04  
## 34 1.88 79.83

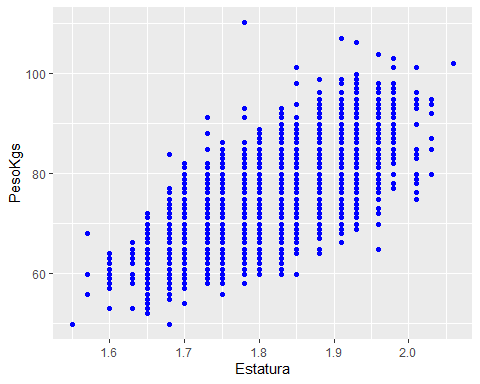
## tail(datos.validacion, 10)

tail(datos.validacion, 10)

## Estatura PesoKgs  
## 17920 1.80 69.85  
## 17923 1.70 59.87  
## 17929 1.88 73.94  
## 17932 1.78 74.84  
## 17937 1.78 67.13  
## 17943 1.91 84.82  
## 17947 1.80 69.85  
## 17949 1.75 71.21  
## 17952 1.91 77.11  
## 17955 1.78 79.83

# Visualizar dispersión de los datos

ggplot(data = datos.entrenamiento) +  
 geom\_point(aes(x = Estatura, y = PesoKgs), col='blue')



# Construir modelos

## Construir modelo regresión lineal simple

modelo.simple <- lm(data = datos.entrenamiento, formula = PesoKgs ~ Estatura)  
  
resumen.mls <- summary(modelo.simple)  
resumen.mls

##   
## Call:  
## lm(formula = PesoKgs ~ Estatura, data = datos.entrenamiento)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -22.064 -2.873 -0.192 2.944 37.497   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) -67.7034 1.1143 -60.76 <2e-16 \*\*\*  
## Estatura 78.8913 0.6144 128.40 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 4.654 on 12535 degrees of freedom  
## Multiple R-squared: 0.5681, Adjusted R-squared: 0.568   
## F-statistic: 1.649e+04 on 1 and 12535 DF, p-value: < 2.2e-16

## Coeficientes a y b

a <- resumen.mls$coefficients[1]  
b <- resumen.mls$coefficients[2]  
a; b

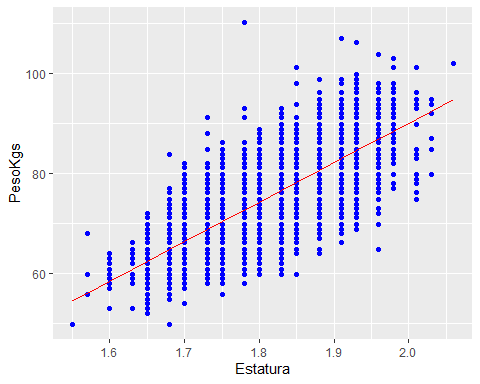
## [1] -67.70338

## [1] 78.89132

## Predecir conforme a fórmula

## Visualizar la recta de regresión lineal

ggplot(data = datos.entrenamiento) +  
 geom\_point(aes(x = Estatura, y = PesoKgs), col='blue') +  
 geom\_line(aes(x = Estatura, y = a + b \* Estatura), col='red')



## Construir modelo regresión polinómico Segunda potencia

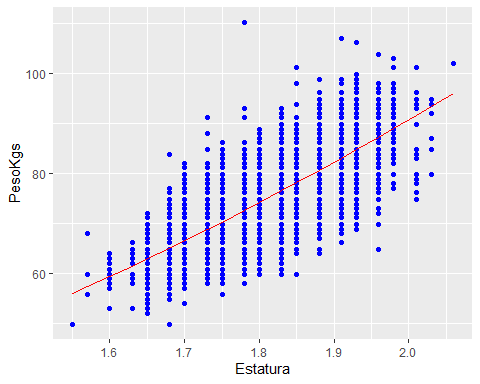
$$
\b\_i \text{ son los coefientes de x}
$$

modelo.poly2 <-lm(data = datos.entrenamiento, formula = PesoKgs ~ poly(Estatura, 2))  
  
resumen.poly2 <- summary(modelo.poly2)  
resumen.poly2

##   
## Call:  
## lm(formula = PesoKgs ~ poly(Estatura, 2), data = datos.entrenamiento)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -22.445 -2.795 -0.152 3.018 37.575   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) 75.27692 0.04155 1811.666 < 2e-16 \*\*\*  
## poly(Estatura, 2)1 597.59922 4.65244 128.449 < 2e-16 \*\*\*  
## poly(Estatura, 2)2 15.36374 4.65244 3.302 0.000962 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 4.652 on 12534 degrees of freedom  
## Multiple R-squared: 0.5684, Adjusted R-squared: 0.5684   
## F-statistic: 8255 on 2 and 12534 DF, p-value: < 2.2e-16

## Curva del modelo Polinómico esegundo nivel

ggplot(data = datos.entrenamiento) +  
 geom\_point(aes(x = Estatura, y = PesoKgs), col='blue') +  
 geom\_line(aes(x = Estatura, y = predict(modelo.poly2, datos.entrenamiento)), col='red')



# Predecir con los dos modelos

Predecir con datos d validación

prediciones.mls <- predict(object = modelo.simple, newdata = datos.validacion)  
  
predicciones.poly2 <- predict(object = modelo.poly2, newdata = datos.validacion)  
  
predicciones <- data.frame(Estatura = datos.validacion$Estatura,  
 PesoKgs = datos.validacion$PesoKgs, prediciones.mls, predicciones.poly2)

## head(predicciones, 10)

head(predicciones, 10)

## Estatura PesoKgs prediciones.mls predicciones.poly2  
## 6 1.73 73.94 68.77860 68.83233  
## 7 1.73 66.22 68.77860 68.83233  
## 11 1.83 79.83 76.66773 76.57058  
## 13 1.88 78.02 80.61230 80.60904  
## 15 1.68 72.12 64.83403 65.13254  
## 24 1.73 69.85 68.77860 68.83233  
## 26 1.78 73.03 72.72317 72.64501  
## 31 1.75 78.93 70.35643 70.34385  
## 33 1.73 68.04 68.77860 68.83233  
## 34 1.88 79.83 80.61230 80.60904

## tail(predicciones, 10)

tail(predicciones, 10)

## Estatura PesoKgs prediciones.mls predicciones.poly2  
## 17920 1.80 69.85 74.30099 74.20169  
## 17923 1.70 59.87 66.41186 66.59891  
## 17929 1.88 73.94 80.61230 80.60904  
## 17932 1.78 74.84 72.72317 72.64501  
## 17937 1.78 67.13 72.72317 72.64501  
## 17943 1.91 84.82 82.97904 83.08631  
## 17947 1.80 69.85 74.30099 74.20169  
## 17949 1.75 71.21 70.35643 70.34385  
## 17952 1.91 77.11 82.97904 83.08631  
## 17955 1.78 79.83 72.72317 72.64501

# Interpretar los modelos

resumen.mls

##   
## Call:  
## lm(formula = PesoKgs ~ Estatura, data = datos.entrenamiento)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -22.064 -2.873 -0.192 2.944 37.497   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) -67.7034 1.1143 -60.76 <2e-16 \*\*\*  
## Estatura 78.8913 0.6144 128.40 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 4.654 on 12535 degrees of freedom  
## Multiple R-squared: 0.5681, Adjusted R-squared: 0.568   
## F-statistic: 1.649e+04 on 1 and 12535 DF, p-value: < 2.2e-16

resumen.poly2

##   
## Call:  
## lm(formula = PesoKgs ~ poly(Estatura, 2), data = datos.entrenamiento)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -22.445 -2.795 -0.152 3.018 37.575   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) 75.27692 0.04155 1811.666 < 2e-16 \*\*\*  
## poly(Estatura, 2)1 597.59922 4.65244 128.449 < 2e-16 \*\*\*  
## poly(Estatura, 2)2 15.36374 4.65244 3.302 0.000962 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 4.652 on 12534 degrees of freedom  
## Multiple R-squared: 0.5684, Adjusted R-squared: 0.5684   
## F-statistic: 8255 on 2 and 12534 DF, p-value: < 2.2e-16